



THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering
Continuous Assessment Test I (2013\2014)
Diploma in Technology (Level 3)
MPZ3231-Engineering Mathematics IA

Duration: One and Quarter ($1\frac{1}{4}$)Hours

RegistrationNumber :

Date: 21st January 2014

Time: 1500 hrs -1615 hrs

Instructions

- Answer All Questions
- Number of pages in the paper - 07.
- All symbols are in standard notation.

1. (a) Given that $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = -2\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$

Find $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$

Hence find the angle between \mathbf{a} and \mathbf{b}

[30%]

(b) \overrightarrow{OP} , \overrightarrow{OQ} and \overrightarrow{OR} are the vectors $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $4\mathbf{i} - \mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 3\mathbf{k}$ respectively. Prove that

- i. OP is perpendicular to PQ and PR .
- ii. The angle between OQ and OR is 60° .
- iii. The area of the triangle PQR is 4.5 square units. [30%]

(c) The position vectors of the points A, B, C and R are given by

$\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \lambda\mathbf{k}$ respectively.

i. Find \overrightarrow{AC} , \overrightarrow{AB} and \overrightarrow{AR} .

ii. Find $\overrightarrow{AR} \cdot (\overrightarrow{AC} \times \overrightarrow{AB})$.

iii. Determine the value of λ such that points A, B, C and R are coplanar.

iv. Find the value of λ such that AR and AC are perpendicular.

[40%]

2. (a) Given that $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$

i. Find x and y so that $A^2 + xI = yA$. where I is the 2×2 unit matrix.

Hence find A^{-1} .

[40%]

ii. Find the value of $\det(A - \lambda I)$.

[20%]

iii. Find the values of λ such that $\det(A - \lambda I) = 0$

[10%]

(b) Consider the following system of equations.

$$3x + y = 10$$

$$7x + 5y = 34$$

i. Write down the above system in matrix form $Ax = B$. [05%]

ii. Using A^{-1} find the solution of the above system. [15%]

iii. Find the adjoint matrix of A and find A^{-1} . [10%]

3. (a) Without expanding the determinants, Prove that

i.
$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$$

ii.
$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$$

[40%]

(b) Consider the system

$$4x + ay = 16$$

$$2ax + 18y = b$$

i. For which values of a , does the system has an unique solution? [30%]

ii. Find those pairs of values (a,b) for which the system has an infinite number of solutions. [40%]

End

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